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Brans–Dicke Cosmological Exact Solution in a Radiation-Filled Robertson–Walker Universe

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In a recent paper Singh and Deo obtained the field equations in Brans-Dicke theory for a radiation-filled universe with Robertson-Walker metric and solved the equations for a particular case. Here we obtain the complete set of solutions of these equations.

1. INTRODUCTION

The field equations in Brans-Dicke theory for a radiation-filled universe with a Robertson-Walker metric have been reduced to the following set equations in a recent paper by singh and Deo (1987):

$$\left(\frac{\dot{Q}}{Q}-\frac{\dot{k}}{k}\right)^2+\frac{K}{Q^2}=-\frac{2\ddot{Q}}{Q}+\frac{\ddot{k}}{k}+\frac{\dot{k}}{k}\left[-\left(1+\frac{\omega}{2}\right)\frac{\dot{k}}{k}+\frac{E}{Q}\right] \tag{1a}$$

$$\left(\frac{\dot{Q}}{Q}-\frac{1}{2}\frac{\dot{k}}{k}\right)^2+\frac{K}{Q^2}=\frac{\dot{k}}{k}\left[\frac{1}{4}\left(1+\frac{2\omega}{3}\right)\frac{\dot{k}}{k}-\frac{E}{Q}\right] \tag{1b}$$

$$\frac{\dot{k}}{k^2}Q^3=-B \tag{1c}$$

where $B=\text{const.}$, and the Robertson-Walker metric is given by

$$ds^2=dt^2-Q^2(t)\left(\frac{dr^2}{1-Kr^2}+r^2\,d\theta^2+r^2\sin^2\theta\,d\phi^2\right) \tag{2}$$

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with $K=+1, 0, -1$ for the space of positive, vanishing, and negative curvature index, respectively, and

$$k=\frac{8\pi}{C^4}\phi^{-1}\left(\frac{4+2\omega}{3+2\omega}\right) \tag{3}$$

where ϕ is the scalar field, ω is the Brans-Dicke constant, and as usual the dot stands for the derivative with respect to t.

Equations (1a)-(1c) were solved by Singh and Deo (1987) for the particular case of

$$Q=dt^n$$

where d is a constant and n is a positive integer. It turns out that the solutions exist only if $n=1$, i.e., for the solutions of Singh and Deo (1987), Q is of the form $Q=dt$, where d is a constant. In the present paper we seek to generalize the work of Singh and Deo by obtaining the complete set of solutions of equations (1a)-(1c).

2. SOLUTIONS

Putting

$$R=\frac{k}{Q^2} \tag{4a}$$

and

$$q=-\ln k \tag{4b}$$

and treating R as a function of q and q as a function of t, one can rewrite equations (1a)-(1c) as

$$\frac{B^2}{4}R_q^2+K=B^2RR_{qq}-\left(\frac{3+2\omega}{4}\right)B^2R^2-EBR \tag{1a$'$}$$

$$\frac{B^2}{4}R_q^2+K=\left(\frac{3+2\omega}{12}\right)B^2R^2+EBR \tag{1b$'$}$$

$$\dot{q}=BR^{3/2}e^{q/2} \tag{1c$'$}$$

Case I. $R\neq$ const. Differentiating (1b′) with respect to R, one finds

$$\frac{B^2}{2}R_{qq}=\frac{3+2\omega}{6}B^2R+EB$$

The above equation and equation (1b′) together readily give (1a′). So for this case (1a′) is a consequence of (1b′) and hence one is really left with equations (1b′) and (1c′) which can be integrated as

$$\int \frac{dR}{[\frac{1}{3}(3+2\omega)R^2+4ER/B-4K/B^2]^{1/2}}=q \tag{5a}$$

and

$$\int \frac{e^{-q/2}\,dq}{BR^{3/2}}=t \tag{5b}$$

Equation (5a) gives R as a function of q; then equation (5b) gives q as a function of t. Then Q and k can be determined from equation (4). Explicit integration of (5a) is dependent on the values of ω, E, B, and K. The various cases are given below.

Case Ia. $(3+2\omega)/3=0$. From (5a),

$$\left(\frac{BR}{E}-\frac{K}{E^2}\right)^{1/2}=q+C_1 \tag{6a}$$

Case Ib. $(3+2\omega)/3>0$. From (5a),

$$\frac{1}{\left[\dfrac{(3+2\omega)}{3}\right]^{1/2}}\log\left\{\left(R+\frac{6E}{B(3+2\omega)}\right)+\left[R^2+\frac{12E}{B(3+2\omega)}R-\frac{12K}{B^2(3+2\omega)}\right]^{1/2}\right\}$$
$$=q+C_2 \tag{6b}$$

Case Ic. $(3+2\omega)/3<0$. From (5a),

$$\frac{1}{[-(3+2\omega)/3]^{1/2}}\sin^{-1}\frac{R+[6E/B(3+2\omega)]}{(4K/B^2)[3/(3+2\omega)]+(4E^2/B^2)[3/(3+2\omega)]^2}$$
$$=q+C_3 \tag{6c}$$

where C_1, C_2, and C_3 are integrating constants.

Case II. $R=\text{const}$. In this case the solution of equation (1′) is trivial. Solving (1′), using (4), and suitably choosing the origin of t, one gets

$$Q\propto t$$

which is the case considered by Singh and Deo (1987).

3. CONCLUSION

We have obtained the complete set of solutions for the field equations in Brans–Dicke theory for a radiation-filled universe given by equations

(1a)–(1c) and metric given by equation (2). The complete solution consists of two different solutions. The first one is given by equations (4a), (4b) and (5a), (5b). The explicit form of the integrals of equations (5a), (5b) are given by equations (6a)–(6c).

The other solution is that obtained by Singh and Deo (1987) and hence need not be discussed.

REFERENCE

Singh, R. T., and Deo, S. (1987). *International Journal of Theoretical Physics*, **26**, 901.